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Inverse neutrinoless double beta decay (and other $\Delta L = 2$ processes)[1]

David London[2]

Laboratoire René J.-A. Lévesque, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

Abstract. I review the prospects for the detection of $\Delta L = 2$ processes at future colliders. Except in contrived models, the process $e^-e^- \to W^-W^-$ is unobservable at future linear colliders unless $\sqrt{s} \gtrsim 2$ TeV, due to constraints from neutrinoless double beta decay. As there are no analogous constraints on the Majorana mass of the ν_μ, $\mu^-\mu^- \to W^-W^-$ could be observed at a muon collider with considerably lower $\sqrt{s}$. One can also consider esoteric processes such as $\gamma\gamma \to \mu^+\mu^+W^-W^-$. Such processes may be observable if $\sqrt{s} \gtrsim 4$ TeV.

There have been several talks at this conference dealing with neutrinoless double beta decay ($\beta\beta_{0\nu}$). In this decay the fundamental $\Delta L = 2$ process is $W^-W^- \to e^-e^-$, mediated by a Majorana neutrino. However, one can turn this around, and consider $e^-e^- \to W^-W^-$ [1, 2, 3]. For obvious reasons, this process is often referred to as "inverse neutrinoless double beta decay," and it can in principle be explored at a future e^+e^- linear collider (NLC) which is run in e^-e^- mode.

The diagrams contributing to $e^-e^- \to W^-W^-$ are shown in Fig. 1. The process is mediated by the exchange of a (gauge-eigenstate) ν_e. However, since the ν_e can mix with other neutrinos,

$$\nu_e = \sum_i U_{ei} N_i \ , \tag{1}$$

the mass eigenstates N_i may be heavy or light, and all of these states will contribute to $e^-e^- \to W^-W^-$.

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[2] e-mail: london@lps.umontreal.ca

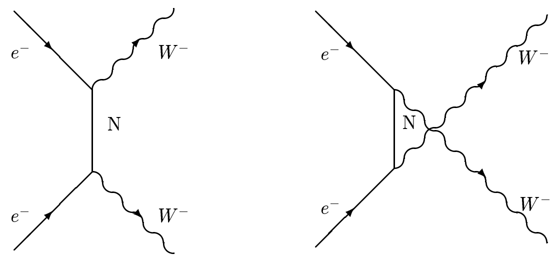

Figure 1. Diagrams contributing to $e^-e^- \to W^-W^-$.

In the limit of $\sqrt{s} \gg M_W$, which is a good approximation for the NLC, the cross section for $e^-e^- \to W^-W^-$ can be written [2]

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\pi} \left(\sum_i M_i \left(U_{ei}\right)^2 \left[\frac{t}{(t - M_i^2)} + \frac{u}{(u - M_i^2)} \right] \right)^2 . \tag{2}$$

There are two interesting limits to consider, which will be important in what follows.

- $s \gg M_i^2$:

$$\sigma = \frac{G_F^2}{4\pi} \left(\sum_i M_i \left(U_{ei}\right)^2 \right)^2 . \tag{3}$$

- $M_i^2 \gg s$:

$$\sigma = \frac{G_F^2}{16\pi} s^2 \left(\sum_i \frac{\left(U_{ei}\right)^2}{M_i} \right)^2 . \tag{4}$$

From the above, we see that the two parameters entering into the $e^-e^- \to W^-W^-$ cross section are $\left(U_{ei}\right)^2$ and M_i. What are the constraints on these quantities? First, the results from low-energy experiments imply that [4]

$$\sum_{i \neq e} |U_{ei}|^2 \leq 6.6 \times 10^{-3} . \tag{5}$$

Second, we can consider unitarity. From Eq. (3) we see that the cross section does not vanish in the limit $\sqrt{s} \to \infty$, violating unitarity. We therefore conclude that either (a)

$$\sum_i M_i \left(U_{ei}\right)^2 = 0 , \tag{6}$$

or (b) there are additional contributions to the process, such as the s-channel exchange of a Δ^{--}. For the moment, let us assume that there are no doubly-charged Higgs bosons. Is Eq. (6) satisfied? The answer to this question is *yes*. It is straightforward to show that

$$\sum_i (U_{ei})^2 M_i = M_{ee}^* \ , \tag{7}$$

where M_{ee} is the Majorana mass of the ν_e. Note that such a mass can only arise in the presence of a Higgs triplet. However, since we have assumed that there are no doubly-charged Higgses, this implies that there are no Higgs triplets. We therefore conclude that $M_{ee} = 0$, so that Eq. (6) is automatically satisfied.

As an explicit example, consider the well-known seesaw mechanism. One adds a right-handed neutrino N to the spectrum, and allows it to acquire a Majorana mass via a Higgs singlet. The mass matrix then looks like

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}, \tag{8}$$

where m is light ($m \sim m_e$) and M is heavy. The two mass eigenstates are N_1 and N_2, with masses $-m^2/M$ and M, respectively. We can therefore write

$$\nu_e = N_1 \cos\theta + N_2 \sin\theta \ , \tag{9}$$

where $\sin\theta = m/M$ ($\cos\theta \simeq 1$). With these masses and mixings, one can clearly see that the relation in Eq. 6 is satisfied. I will return to this example later, but one key point to retain is that the mixing naturally satisfies $U_{ei} \sim m_e/M_i$.

From the above discussion, we therefore conclude that unitarity puts no constraints on $(U_{ei})^2$ and M_i. However, there are constraints from $\beta\beta_{0\nu}$. In $\beta\beta_{0\nu}$, the relevant scale is the mass of the proton. For the N_i which are light, $M_i \ll 1$ GeV, measurements of $\beta\beta_{0\nu}$ restrict [5]

$$\langle m_\nu \rangle = \sum_{i \ light} (U_{ei})^2 M_i < 0.62 \text{ eV} \ . \tag{10}$$

On the other hand, for heavy neutrinos, $M_i \gg 1$ GeV, the relevant quantity is $\sum_{i \ heavy} (U_{ei})^2 / M_i$, with the constraint [5]

$$\sum_{i \ heavy} (U_{ei})^2 \frac{1}{M_i} < 1.0 \times 10^{-4} \text{ TeV}^{-1} \ . \tag{11}$$

How does this all affect inverse $\beta\beta_{0\nu}$? Suppose, first, that all the neutrinos are light. Combining the expression for the cross section [Eq. (3)] with the constraint in Eq. (10), we find

$$\sigma(e^- e^- \to W^- W^-) \lesssim 10^{-17} \ fb \ . \tag{12}$$

This is clearly unobservable. Let us now suppose that all the neutrinos are heavy, i.e. $M_i \gg \sqrt{s}$. For $\sqrt{s} = 1$ TeV, from Eqs. (4) and (11) we find

$$\sigma(e^- e^- \to W^- W^-) < 0.01\ fb\ . \tag{13}$$

For an NLC luminosity of 80 fb^{-1}, this yields only 0.8 events/year, which is not promising. Furthermore, by the time the NLC is built, the limits from $\beta\beta_{0\nu}$ will be much stronger. One must therefore conclude that, due to low-energy constraints from $\beta\beta_{0\nu}$, the process $e^- e^- \to W^- W^-$ at a 1 TeV NLC will be unobservable.

But this raises a question: can the constraints from $\beta\beta_{0\nu}$ be evaded? For example, consider the case of one heavy neutrino ($M \sim 1$ TeV) with mixing $U_{ei}^2 = 5 \times 10^{-3}$. Ignoring $\beta\beta_{0\nu}$, this gives an $e^- e^- \to W^- W^-$ cross section at $\sqrt{s} = 1$ TeV of 10 fb. This is enormous: it would yield 800 events/year. Unfortunately, these mass and mixing parameters give $(U_{ei})^2/M = 5 \times 10^{-3}$ TeV^{-1}, in violation of the constraints from $\beta\beta_{0\nu}$ [Eq. (11)]. However, couldn't we add other heavy neutrinos, whose factors of $(U_{ei})^2/M$ cancelled this? The answer is obviously *yes*. For example, we could add one neutrino of mass $M = 100$ GeV, with mixing $U^2 = -5\times 10^{-4}$. Or we could add 10 neutrinos of mass $M = 10$ TeV, with mixings $U^2 = -5 \times 10^{-3}$. In fact, there are an infinite number of possibilities.

There are, however, a couple of problems with such scenarios. First, they are all somewhat contrived, perhaps even fine-tuned. Second, recall the seesaw mechanism I described earlier. Based on that example, we would expect that $U_{ei} \sim m_e/M_i$. However, this is not obeyed in the above scenarios: in these cases the mixing of the heavier neutrinos is larger than, or equal to that of the lighter neutrinos. What we conclude from these examples is that, although it is possible to evade bounds from $\beta\beta_{0\nu}$ — there are regions of parameter space where cancellations occur [3] — it does not happen naturally. There is one more point to be made here: in the scenario with a lighter neutrino of mass $M = 100$ GeV, the N would first be discovered in $e^+ e^- \to N\nu$, and we would be able to verify that it is indeed a Majorana neutrino. Its mass and mixing would be measured, and we would realize that it violated the $\beta\beta_{0\nu}$ bound. We would therefore *infer* the existence of heavier neutrinos before they were discovered.

To summarize the above discussion: it is possible to evade the constraints from $\beta\beta_{0\nu}$, but one requires rather somewhat unnatural scenarios. If we do not allow such solutions, we can calculate the discovery limit for $e^- e^- \to W^- W^-$ at the NLC, as a function of U_{ei}^2 and M_i, for various values of $\sqrt{s}$. The results are shown in Fig. 2, where we have assumed a luminosity of $80(\sqrt{s}/(1\ \mathrm{TeV}))^2\ fb^{-1}$ and demanded 10 events for discovery[3]. From this figure we see that constraints from $\beta\beta_{0\nu}$ essentially rule

[3] In this figure, taken from Ref. [2], the constraint from $\beta\beta_{0\nu}$ has been taken to be

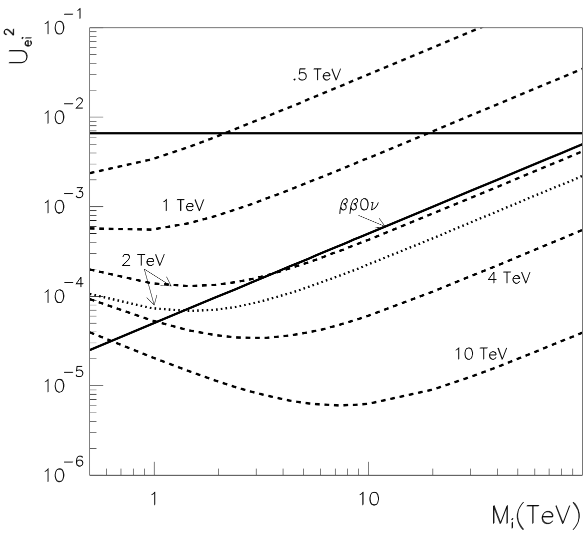

Figure 2. Discovery limit for $e^-e^- \to W^-W^-$ at the NLC as a function of M_i and $(U_{ei})^2$ for various values of $\sqrt{s}$ (dashed lines). In all cases, the parameter space above the line corresponds to observable events. We also superimpose the experimental limit from $\beta\beta_{0\nu}$ (diagonal solid line), as well as the limit on $(U_{ei})^2$ (horizontal solid line). Here, the parameter space above the line is ruled out.

out the observation of the process $e^-e^- \to W^-W^-$ at an NLC of $\sqrt{s} = 2$ TeV or less. For a 4 TeV or 10 TeV NLC, there exists a sizeable region of M_i-$(U_{ei})^2$ parameter space, not ruled out by $\beta\beta_{0\nu}$, which produces an observable signal for $e^-e^- \to W^-W^-$.

So far, the entire discussion has taken place assuming that there is no doubly-charged Higgs boson. Let us now return to the beginning and assume that there is a Δ^{--} which contributes to $e^-e^- \to W^-W^-$. How does this change things? In fact, it does not change the conclusions at all. To see this, consider the left-right symmetric model. This model contains a Δ^{--}, which is a member of a representation in which the neutral partner obtains a vacuum expectation value and gives mass to both the W and ν_e. The Δ^{--} contribution to $e^-e^- \to W^-W^-$ involves the product of $g_{\Delta ee}$ and $g_{\Delta WW}$, which are the couplings of the Δ^{--} to e^-e^- and W^-W^-,

$\sum_{i\ heavy} (U_{ei})^2 \frac{1}{M_i} < 0.7 \times 10^{-4}$ TeV^{-1}, which is slightly stronger than that given in Eq. (11).

respectively. However, direct calculation gives

$$g_{\Delta ee} g_{\Delta WW} \propto g^2 \frac{M_{ee}}{M_\Delta} \ , \tag{14}$$

where M_{ee} is the Majorana mass of the ν_e. However, from Eq. (10), we have $M_{ee} \lesssim 1$ eV (since $U_{ee} \simeq 1$), so that the product $g_{\Delta ee} g_{\Delta WW}$ is tiny. Thus, the exchange of the Δ^{--}, though necessary for unitarity, does not affect the cross sections, and so the previous conclusions hold. (In fact, if the seesaw mechanism is used, the process $e^- e^- \to W^- W^-$ is completely unobservable, as per Eq. (12).)

Since the prospects for the observation of $e^- e^- \to W^- W^-$ are relatively poor, the obvious next question is: what about other $\Delta L = 2$ processes? The key observation is that, although $\beta\beta_{0\nu}$ strongly constrains the Majorana mass of the ν_e, ν_μ (and ν_τ) are not similarly constrained. Therefore it stands to reason that one should consider $\Delta L = 2$ processes involving ν_μ.

First, there has been much discussion recently about the prospects for building a linear muon collider. If such a collider can be built, then it can be run in $\mu^- \mu^-$ mode, and one can look for the process $\mu^- \mu^- \to W^- W^-$. In this case, the only constraint is on ν_μ mixing [4]:

$$\sum_{i \neq \mu} |U_{\mu i}|^2 \leq 6.0 \times 10^{-3} \ . \tag{15}$$

The cross section is therefore quite sizeable, even at $\sqrt{s} = 500$ GeV. (To see this, simply refer to Fig. 2, ignoring the constraints from $\beta\beta_{0\nu}$).

Second, one can consider more esoteric possibilities, such as $\gamma\gamma \to \mu^+ \mu^+ W^- W^-$ [2]. The cross section for this process is shown in Fig. 3. Assuming a luminosity of $80(\sqrt{s}/(1 \text{ TeV}))^2 \ fb^{-1}$, one sees from this figure that $\gamma\gamma \to \mu^+ \mu^+ W^- W^-$ requires $\sqrt{s} \gtrsim 4$ TeV for observability. (Note, however, that if $M_i < \sqrt{s}$, the new neutrino is far more likely to be first discovered via single production in $e^+ e^- \to \nu_\mu N_i$ than in $\gamma\gamma \to \mu^+ \mu^+ W^- W^-$.)

Finally, there are other, even more exotic processes, such as $e^- \gamma \to \nu_e \mu^- \mu^- W^+$ [2]. However, the cross sections for such processes are even smaller than those for $\gamma\gamma \to \mu^+ \mu^+ W^- W^-$.

To conclude: if a linear $e^+ e^-$ collider is ever built, it can in principle be run in $e^- e^-$ mode in order to search for the $\Delta L = 2$ process $e^- e^- \to W^- W^-$. However, except in rather contrived scenarios, constraints from low-energy neutrinoless double beta decay effectively rule out this process unless $\sqrt{s} > 2$ TeV. Of course, by the time such a collider is built, the constraints from $\beta\beta_{0\nu}$ will probably be considerably more stringent, so that even higher centre-of-mass energies will be required.

As there are no analogous constraints on the Majorana mass of the ν_μ, one can consider $\Delta L = 2$ processes involving a ν_μ. For example, should

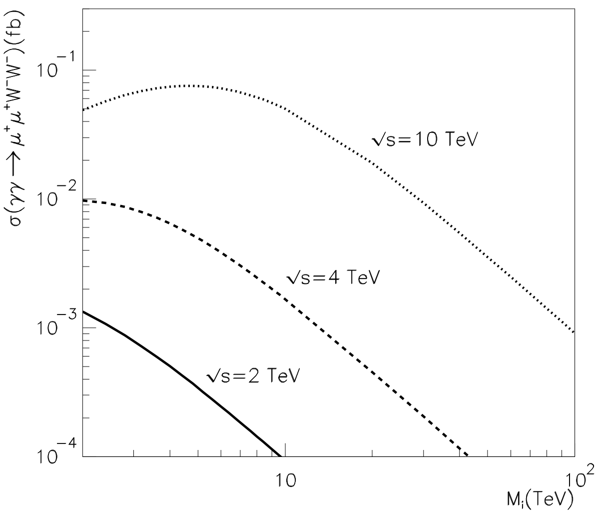

Figure 3. Cross section for $\gamma\gamma \to \mu^+\mu^+W^-W^-$ at the NLC assuming $(U_{\mu i})^2 = 6.0 \times 10^{-3}$ for $\sqrt{s} = 2$ TeV (solid line), 4 TeV (dashed line) and 10 TeV (dotted line).

a muon collider be built, the process $\mu^-\mu^- \to W^-W^-$ could be readily observable, even for $\sqrt{s} = 500$ GeV. In the absence of such a collider, one must consider more esoteric processes such as $\gamma\gamma \to \mu^+\mu^+W^-W^-$. These are observable for $\sqrt{s} \gtrsim 4$ TeV.

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